DECOMPOSITION METHODS FOR OPTIMAL CONTROL OF TRANSPORT

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1. Introduction

In many practical transport problems optimization techniques are used to get effective vehicle routes satisfying demands at network nodes[3]. However, there are problems which can't be solved by these techniques due to their size. Such situations arise especially in the cases when operative control is used, eg. in the cases when input data are known only short time before output routes have to be given to drivers. For this sort of control mainly PC computers are used but their memory capacity and computational speed impose strong constraint on solvable problem size. There is a way how to handle this difficulties. The way is based on decomposition of the original problem to a number of smaller subproblems solvable by mentioned computers.

This paper considers two applications of the decomposition method. The first solves Period Vehicle Routing Problems [2] when single node demands can be satisfied by several ways (combinations of working days during considered season). The demand decomposition into single days of the season serves here as pre-optimization method which creates subproblems solvable as single Vehicle Routing Problems [3] for appropriate days. The second application deals with reducing the computational time of the single Vehicle Routing Problem algorithm when the problem in a large network is solved.

2. Mathematical formulations of the problems

Let us consider basic problem, so called Vehicle Routing Problem solution of which determines daily task (route) for vehicle drivers of some road transport system. We assume a network consists of nodes $1,\dots,n$ where node 1 is a depot and nodes from a set $N^t-\{1\}$ are customers whose demands b_i should be satisfied on the given day t.

Let matrix $\{c_{ij}\}$ of the shortest distances between each two network nodes is given and let K_d signify capacity of the vehicle d from a set D^t.

The problem is to design the routes of the vehicles with the minimum total length so that each customer will be visited just once and least vehicle capacity should be exceeded for any vehicle (route) d. All the vehicle routes should begin and end at the node 1. For each day t the validity of $\sum_{d\in D^t} K_d \geq \sum_{i\in N^t} b_i$ is assumed.

To establish the model we shall use non-negative integer variables x_{ij}^d which get value 1 if the vehicle d moves from node i to node j and get value 0 in the opposite case. The problem can be formulated in this way:

$$\text{minimize} \sum_{d\in D^t} \sum_{i\in N^t} \sum_{j\in N^t} c_{ij} x_{ij}^d \qquad (1)$$

$$\text{subject to} \sum_{d\in D^t} \sum_{i\in N^t} x_{ij}^d = 1 \text{ for } j\in N^t-\{1\} \qquad (2)$$

$$\sum_{d\in D^t} \sum_{i\in N^t} x_{ij}^d = \sum_{d\in D^t} \sum_{i\in N^t} x_{ji}^d \text{ for } j\in N^t \qquad (3)$$

$$\sum_{i\in N^t} b_i \sum_{j\in N^t} x_{ij}^d \geq K_d \quad \text{for } d\in D^t \qquad (4)$$

$$\sum_{i\in w} \sum_{j\in w} x_{ij}^d \geq |w|-1 \text{ for } d\in D^t \text{ and} \qquad (5)$$

for each subset $w\subseteq N^t-\{1\}$.

Constraints (2) and (3) express that whenever a vehicle enters a node it leaves that node, and each node will be visited just once. Inequalities (4) ensure that the capacity of a vehicle will not be exceeded. Inequalities (5) state that each route will start and finish at the depot.

There are many methods [3],[6],[7] to solve the problem. The problem (1)-(5) will be more complicated if we consider a period T of several days and if a customer i requi-

res k visits during the period while these visits may only occur in one of a given set S_i of allowable combinations [2]. For example see Fig.1 where three-day period is considered. The days are distinquished by full, dashed and dotted lines. According to the Fig. 1 the customer 2 must be visited on "full" day (he has only one allowable combination), the customer 3 must be visited either on "full" and "dashed" day or on "full" and "dotted" day. The customer 4 will be visited only once during the period but he has three allowable combinations.

Fig. 1

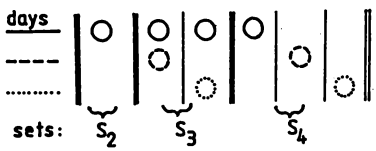
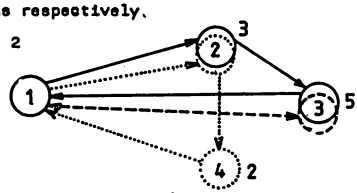


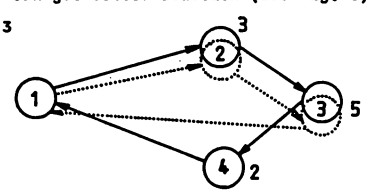
Fig.2 shows an admissible solution of Period Vehicle Routing Problem for vehicle capacity equal to 10 and for average customer demands written outside the node in the figure. We can see that three routes were used during the period in the case when customers 3 and 4 had demanded their first and third combinations respectively.

Fig. 2



Using another combinations (the second for the customer 3 and the first for the customer 4) we can get better solution (see Fig. 3).

Fig. 3



Let S be a set of all combinations allowable at least for one customer of the network. Let $\{a_{kt}\}$ be an incidence matrix where the rows correspond to the combinations from S and columns to the days from the period T. The equality $a_{kt}=1$ holds if k-th combination contains day t, otherwise $a_{kt}=0$. We denote y_{ik} an integer non-negative variable which gets value 1 if the demands of the customer i should be satisfied on the days of combination k, $y_{ik}=0$ otherwise.

Let $F(N^t)$ be optimum objective value of the problem (1)-(5). Thus we can state the following model:

$$\text{minimize} \quad \sum_{t\in T} F(N^t) \qquad (6)$$

$$\text{subject to} \quad \sum_{k\in S_i} y_{ik}=1 \quad \text{for } i=1,\ldots,n \qquad (7)$$

$$\sum_{i=1}^{n} \sum_{k\in S_i} b_i a_{kt} y_{ik} \le \sum_{d\in D^t} k_d \quad \text{for } t\in T \qquad (8)$$

$$N^t=\{1\}\cup\{i/\sum_{k\in S} a_{kt}y_{ik}=1\} \quad \text{for } t\in T \qquad (9)$$

Constraints (7) express that just one allowable combination must be chosen for each customer i. Constraints (8) state that the daily capacity of the fleet D^t must not be exceeded and the constraints (9) define sets N^t of customers which are "active" on the day t. The problem (6)-(9) can be formulated as integer-linear program [2], [4], [5] but it is so large that it can't be solved due to excesive computational time. This is why an approximation of the $F(N^t)$ is used which doesn't involve vehicle routes and then problem (6)-(9) is solved apart from the problems (1)-(5). We meet similar difficulties when we design the decomposition of the problem (1)-(5) where $N^t=\{1,\ldots,n\}$ is assumed. In this case we want to divide the node set N^t to q parts W_k so that $\bigcup_{k=1}^{q} W_k=\{1\}$ will hold and last the total demand of customers from W_k will be greater than some amount Q. The total length of the vehicle routes should be minimum. We denote z_{ik} an integer non-negative variable which gets value 1 if the node i is assigned to the set W_k, $z_{ik}=0$ otherwise.

Thus we get the model:

$$\text{minimize} \quad \sum_{k=1}^{q} F(W_k) \qquad (10)$$

$$\text{subject to} \quad \sum_{k=1}^{q} z_{ik}=1 \quad \text{for } i=2,\ldots,n \qquad (11)$$

$$\sum_{i=2}^{n} b_i z_{ik} \le Q \quad \text{for } k=1,\ldots,q \qquad (12)$$

3. Heuristics

To solve the problem (6)-(9) we used method published in [2] and developed several modifications of it. The frame of the algorithms consists of the following steps:

1. Order the set of customers according to given criterion
2. Select arbitrary |T| candidates for cen-

tres from the nodes of the network. Denote the set of candidates as C.

3. Denote S_i set of allowable combinations of the customer i, $|S_i|$ number of elements of S_i. Let I_j be set of $|S_i|$ nearest customers to the centre $j \in C$.
   From matrix
   $$m_{jt} = \sum_{i \in I_j} \left(\sum_{k \in S_i} a_{kt} \right) / |S_i|$$
   and solve the assignment (maximization) problem for the matrix to determine centre from C for each day $t \in T$.
4. Assign some combination $k \in S_i$ to each customer i according to given criterion.
5. Recompute centre j for each day t to minimize given criterion and reform the set C.
6. Go through the set of all pairs of customers (i,j) where $i<j$ and test each pair of combinations (k,f) where $k \in S_i$, $f \in S_j$. Choose the best pair of combinations according to given criterion and change the assignment of combinations to customers if necessary.
7. Recompute centres as in step 5 and if there is some possibility to improve the solution go to step 6.

We derived algorithms JAN, JAF, JAC, JAD and JAB in the following way.

JAN - it is original algorithm [2] where customers are ordered towards increasing $|S_i|$ in the step 1. The criterion at the steps 4 and 6 is $\sum_{t \in T} c_{ij_t} a_{kt}$ and the criterion at the step 5 is $\sum_{i=2}^{n} c_{ij} a_{k_i t}$.

JAF - Customers are ordered towards decreasing value of $\sum_{k \in S_i} \sum_{t \in T} a_{kt} / |S_i|$ at the step 1.

JAC - Customers are ordered towards decreasing value of c_{1i} at the step 1.

JAD - it is the same algorithm as JAN but the steps 6 and 7 are repeated two times.

JAB - the criterion at the steps 4 and 6 is $\sum_{t \in T} b_i c_{ij_t} a_{kt}$ and the criterion at the step 5 is $\sum_{i=2}^{n} b_i c_{ij} a_{k_i t}$

Computational results are given in the next section. To solve the problem (10)-(12), two heuristics were developed and tested [4], [5].

Both heuristics can be divided into two parts. In the first part the set K of q special nodes (so called "seeds") is determined according to various criteria. The second part is the same for both procedures. Each seed k formes initial node set of the component W_k here. Taking each seed k one by one in the cycle, the component W_k is enlarged by one node at every step of the part. The node i is chosen from the set of "free" nodes so that value $c_{1i}+r.c_{ik}$ will be minimum for a shape parameter r.

The first part of the first heuristic [9] uses the following rule to create the set K of the seeds:

a) Select j for expression $c_{1j} + vb_j$ to be maximum.

b) Find j for expression $(c_{1j} + vb_j) \cdot (\text{II } c_{ji} + vb_i))$ to be maximum and set $K = K \cup \{j\}$ $^{i \in K}$.

c) Repeat b) until $|K| = q$.

(The parameter v is some "weight" of the demand.) The first part of the second heuristic [1] uses the same rule but it uses value m_j instead of single demand b_j. The m_j is defined by

$m_j = \sum_{i \in c_j} b_j$ where c_j is a cluster of the first m nodes of the sequence ordered towards decreasing value of $c_{1i} + c_{ij}$.

4. Computational experiments

All algorithms were programmed using FORTRAN H compiler and computer EC 1045. The algorithms JAN, JAF, JAC, JAD and JAB were tested on data of 840 node network for 5 day period with daily fleet capacity 19000, 17500 and 16000 elements per day. The data were taken from real 5 day beer delivery proolem of the brewery Nošovice. After the decomposition the routes for each day of the period were computed and the results were compared with the original assignment of the combinations to the customers.

The results are summarized in Tab. 1

Tab. 1

Algorithm	Daily fleet capacity	Total length [km]	Savings [%]	
JAN	19000	13376	5.3	6.2
	17500	13175	6.7	
	16000	13197	6.5	
JAF	19000	13486	4.5	5.1
	17500	13148	6.9	
	16000	13559	4.0	
JAC	19000	13194	6.6	5.8
	17500	13192	6.6	
	16000	13499	4.4	
JAD	19000	13263	6.1	6.4
	17500	13060	7.5	
	16000	13311	5.7	
JAB	19000	13158	6.8	7.4
	17500	13022	7.8	
	16000	13050	7.6	
Original result		14120		

The heuristics for the problem (10) - (12) were compared [8] using the daily situations of the beer delivery problem and the second [1] heuristic proved to be better. The computational times are shown in Tab. 2 for individual days of the period and for various number q of components determined by the heuristic. To solve vehicle routing problems in these cases, a simple and fast algorithm was used. The effect of a number n of network nodes on the computational time T(n) was approximately $T(n) = -0.16 + 0.02\,(n)^{1.3}$ for $n \geq 30$.

day:		1	2	3	4	5
number of nodes:		217	274	230	264	160
comp. time [s] for q	q 1	20.0	30.5	24.5	28.4	13.6
	2	16.1	24.1	20.4	23.7	10.9
	4	11.8	20.5	16.5	18.4	8.4

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